16-10-2007

Diversity Performance of Precoded OFDM with MMSE Equalization

Xiaojing Huang
University of Wollongong, huang@uow.edu.au

Diversity Performance of Precoded OFDM with MMSE Equalization

Abstract

Two sets of bit error rate (BER) lower bounds for precoded orthogonal frequency division multiplexing (OFDM) systems using minimum mean square error (MMSE) equalization over frequency-selective multipath fading channels are evaluated by Monte Carlo method in this paper. The first set represents the best performance under different data group sizes used for precoding, whereas the second set represents the best performance under different channel multipath diversity orders. These performance bounds can serve as the guidelines for system designers to decide proper data group sizes for precoded OFDM systems in order to achieve better trade-off between system performance and complexity. Numerical results also confirm the usefulness of these lower bounds for practical OFDM systems.

Disciplines

Physical Sciences and Mathematics

Publication Details

This conference paper was originally published as Huang, X, Diversity Performance of Precoded OFDM with MMSE Equalization, 7th International Symposium on Communications and Information Technologies ISCIT 2007, Sydney, 16-19 Oct, 802-807.

Diversity Performance of Precoded OFDM with MMSE Equalization

Xiaojing Huang
School of Electrical, Computer and Telecommunications Engineering
University of Wollongong, Northfields Avenue, Wollongong, NSW 2522, Australia
Tel: +61-2-4221-4610, Email: huang@uow.edu.au

Abstract—**Two sets of bit error rate (BER) lower bounds for precoded orthogonal frequency division multiplexing (OFDM) systems using minimum mean square error (MMSE) equalization over frequency-selective multipath fading channels are evaluated by Monte Carlo method in this paper. The first set represents the best performance under different data group sizes used for precoding, whereas the second set represents the best performance under different channel multipath diversity orders. These performance bounds can serve as the guidelines for system designers to decide proper data group sizes for precoded OFDM systems in order to achieve better trade-off between system performance and complexity. Numerical results also confirm the usefulness of these lower bounds for practical OFDM systems.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is a data transmission scheme which modulates data symbols in parallel on orthogonal subcarriers [1,2]. With OFDM, the effect of intersymbol interference (ISI) caused by channel time spread can be easily mitigated. An OFDM transmitter can be implemented by the inverse fast Fourier transform (IFFT) with cyclic prefix (CP) insertion or zero-padded suffix (ZP) appending, and simple frequency domain channel equalization can be applied at the receiver via fast Fourier transform (FFT). Due to these advantages, OFDM has been widely used in today's digital communication systems such as wireless personal/local/metropolitan area networks (WPANs/ WLANs/WMANs) and digital audio/video broadcasting services (DAB/DVB) [3-6]. It is also a candidate for future generation wireless mobile communication systems [7].

However, the conventional OFDM systems suffer from some major disadvantages and considerable research has been undertaken to overcome them over the past decades. First, the transmitted signal waveform has a large peak-to-average power ratio (PAPR), which reduces the power efficiency of the OFDM systems [8-11]. Second, the receiver performance is sensitive to carrier frequency offset which causes inter-carrier interference (ICI). Thus, complicated frequency synchronization is necessary [12,13]. Third, the uncoded OFDM system only achieves diversity order one and hence performs poorly in frequency-selective channels. Channel coding has been traditionally used to improve the diversity across frequency and time [14,15], and recently linear precoding and block spreading for OFDM systems are introduced to improve the frequency diversity performance [16-22].

Precoded OFDM divides a block of modulated data to be transmitted in an OFDM symbol into groups and applies a unitary matrix to each data group to obtain different linear combinations of the data symbols. After subcarrier mapping, the data symbols are spread across the transmission frequency band. Thus, if a subcarrier experiences a deep fade after transmitting over a frequency-selective multipath channel, the data symbol can be still recovered from other received subcarriers so that the system performance is improved due to the increased diversity order.

There are mainly two factors which determine the performance of a precoded OFDM system. One is the equalization/detection method used at the receiver. The other is the precoding data group size. Regarding the equalization/detection method, the maximum-likelihood (ML) detection offers better performance than other linear equalization techniques such as zero-forcing (ZF) and minimum mean square error (MMSE) equalizations. However, the ML detection requires higher computational complexity especially when the data group size is large. In practice, the linear equalization is preferable.

Intuitively, the larger the data group size is, the better the system performance will be. However, larger data group size also implies higher implementation complexity. On the other hand, if the data group size is too small, the available diversity introduced by the multipath channel can not be fully exploited. Therefore, how to determine a proper data group size according to the available multipath diversity order is of significance to practical OFDM system design.

In this paper, the bit error rate (BER) lower bounds of the MMSE equalization in precoded OFDM systems are evaluated for different data group sizes under the assumption that the channel provides a full diversity, which indicate the best performance the MMSE equalization could achieve for a given data group size. Further, assuming a sufficiently large data group size, the performance lower bounds of the MMSE equalization under different multipath diversity orders are also evaluated, which show the best performance the MMSE equalization could achieve for a given multipath diversity order. These performance bounds can serve as the guidelines for system designers to decide suitable group sizes for precoded OFDM systems in order to achieve the desired performance with affordable complexity.

The rest of the paper is organized as follows. In Section II, the precoded OFDM system models are presented. In Section

This research is supported by Australian Research Council Discovery Project DP0558405.

1-4244-0977-2/07/$25.00

III, the BER of the MMSE equalization is formulated as a function of the data group size and the multipath diversity order. Section IV evaluates the two sets of BER lower bounds using Monte Carlo simulation method. Numerical results of system performance for a precoded OFDM system are also provided to confirm the evaluated bounds and demonstrate the usefulness of these bounds to system design. Finally, conclusions are drawn in Section V.

II. SYSTEM MODELS

Referring to the transmitter model shown in Fig. 1 (a), let $x[i]$, $i = 0,1,\cdots,MN-1$, denote MN data symbols (M and N are integer powers of 2), which are modulated from the information data bits after binary phase shift keying (BPSK), quadrature phase shift keying (QPSK) or any other quadrature amplitude modulation (QAM) constellation mapping. Before precoding, the MN data symbols are firstly divided into N groups of size M with the n th group denoted as a vector

$$\mathbf{x}_n = \left(x[nM], x[nM+1], \cdots, x[nM+M-1]\right)^T, \quad n = 0,1,\cdots,N-1, \qquad (1)$$

where $(\cdot)^T$ denotes matrix transposition, and then expressed as a vector $\mathbf{x} = \begin{pmatrix} \mathbf{x}_0 \\ \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_{N-1} \end{pmatrix}$ after serial-to-parallel conversion (S/P).

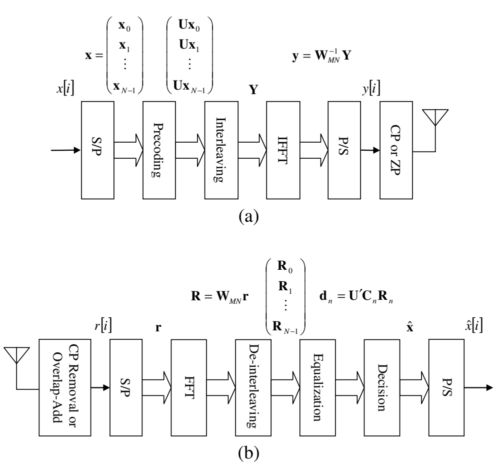

Fig. 1. Precoded OFDM system models: (a) transmitter and (b) receiver. $\mathbf{W}_{MN}^{-1} = \frac{1}{MN}\left(e^{j\frac{2\pi}{MN}nm}\right)_{MN\times MN}$ and $\mathbf{W}_{MN} = \left(e^{-j\frac{2\pi}{MN}nm}\right)_{MN\times MN}$ are the *MN*-point inverse Fourier transform and Fourier transform matrices respectively.

The precoding process is to apply an $M\times M$ unitary matrix $\mathbf{U}$, which satisfies the property $\mathbf{U}\mathbf{U}' = \mathbf{U}'\mathbf{U} = \mathbf{I}$, where $(\cdot)'$ denotes transposition and complex-conjugation operation and $\mathbf{I}$ is the identity matrix of order M, to each vector $\mathbf{x}_n$ to produce a precoded vector where each element is a linear combination of the symbols in vector $\mathbf{x}_n$. To better exploit frequency diversity, the precoded symbols are preferably mapped onto subcarriers equally spaced across the transmitted bandwidth. This is equivalent to a block interleaving operation among N precoded vectors $\mathbf{U}\mathbf{x}_n$, $n = 0,1,\cdots,N-1$, and then performing IFFT of length MN on the resulting precoded and interleaved vector $\mathbf{Y}$.

After IFFT and parallel-to-serial conversion (P/S), a time domain sequence $y[i]$, $i = 0,1,\cdots,MN-1$, is produced. To form a precoded OFDM symbol, either a CP or a ZP of sufficient length (longer than the maximum channel multipath delay in samples) are added to $y[i]$ to avoid interference between adjacent precoded OFDM symbols and turn the linear convolution of the transmitted signal with the channel impulse response into a circular one.

The precoded OFDM signal is then transmitted over a frequency-selective multipath fading channel and received at the receiver baseband. By removing the CP or performing an overlap-add operation, MN -point received precoded OFDM samples $r[i]$, $i = 0,1,\cdots,MN-1$, will be produced. After FFT and de-interleaving, the discrete-time received signal can be expressed in the frequency domain as

$$\mathbf{R}_n = \mathbf{H}_n\mathbf{U}\mathbf{x}_n + \mathbf{V}_n, \quad n = 0,1,\cdots,N-1, \qquad (2)$$

where

$$\mathbf{R}_n = \left(R[n], R[N+n], \ldots, R[(M-1)N+n]\right)^T \qquad (3)$$

is a vector of M elements which are decimated from $R[k]$, the MN -point discrete Fourier transform (DFT) of $r[i]$, by a down-sampling factor N,

$$\mathbf{H}_n = diag\left(H[n], H[N+n], \ldots, H[(M-1)N+n]\right) \qquad (4)$$

is an $M\times M$ diagonal matrix with diagonal elements decimated from $H[k]$, the MN -point DFT of the normalized discrete channel impulse response $h[i]$, and $\mathbf{V}_n$ is a zero-mean Gaussian noise vector with covariance matrix $E\{\mathbf{V}_n\mathbf{V}_n'\} = \sigma_V^2\mathbf{I}$, where $E\{\cdot\}$ denotes ensemble average.

To recover the transmitted data vector $\mathbf{x}_n$, equalization and detection must be performed on the received signal $\mathbf{R}_n$. Due to the complexity of the optimum ML detection, only the MMSE equalization is considered, since it can simply use a one-tap equalizer for each subcarrier in the frequency domain. The equalization and detection process can be described as follows. Let $C[k]$ denote the one-tap equalizer coefficient to be applied to $R[k]$ on the subcarrier k and

$$\mathbf{C}_n = diag(C[n], C[N+n], ..., C[(M-1)N+n]) \quad (5)$$

denote an $M \times M$ diagonal matrix with diagonal elements $C[lN+n]$, $l = 0,1,\cdots,M-1$. First, applying $\mathbf{C}_n$ to $\mathbf{R}_n$ produces the equalized precoded data vector $\mathbf{C}_n\mathbf{R}_n$. Second, using $\mathbf{U}'$ to remove the precoding yields the decision variable vector $\mathbf{d}_n = \mathbf{U}'\mathbf{C}_n\mathbf{R}_n$. Finally, an estimate of the transmitted data vector $\mathbf{x}_n$ is obtained after hard decision. Repeating the above process for $n = 0,1,\cdots,N-1$, all the transmitted data symbols are retrieved.

III. PERFORMANCE OF MMSE EQUALIZATION

We first derive the post-equalization signal-to-noise ratio (SNR) as a function of the equalizer coefficients $C[lN+n]$ for the received signal vector $\mathbf{R}_n$. According to the above described equalization process, the decision variable vector can be expressed as

$$\mathbf{d}_n = \mathbf{U}'\mathbf{C}_n\mathbf{R}_n = \mathbf{U}'\mathbf{C}_n\mathbf{H}_n\mathbf{U}\mathbf{x}_n + \mathbf{U}'\mathbf{C}_n\mathbf{V}_n. \quad (6)$$

Assume that the data symbols in $\mathbf{x}_n$ are independent with average power σ_x^2 so that $E\{\mathbf{x}_n\mathbf{x}_n'\} = \sigma_x^2\mathbf{I}$. The covariance matrix of $\mathbf{d}_n$ can be derived as

$$E\{\mathbf{d}_n\mathbf{d}_n'\} = \sigma_x^2\mathbf{U}'\mathbf{C}_n\mathbf{H}_n\mathbf{H}_n'\mathbf{C}_n'\mathbf{U} + \sigma_V^2\mathbf{U}'\mathbf{C}_n\mathbf{C}_n'\mathbf{U}. \quad (7)$$

Suppose that we want to decide the m th data symbol $x[nM+m]$ in $\mathbf{x}_n$ from the m th element in $\mathbf{d}_n$. The useful signal component can be found from the first term on the right-hand-side of (6) as $\sum_{l=0}^{M-1} C[lN+n]H[lN+n]u_{l,m}\Big|^2 \cdot x[nM+m]$, where $u_{l,m}$ is an element of $\mathbf{U}$ at the l th row and the m th column, and thus the useful signal power after equalization is

$$\left|\sum_{l=0}^{M-1} C[lN+n]H[lN+n]u_{l,m}\right|^2 \Bigg|^2 \sigma_x^2 = q_0[n,m]. \quad (8)$$

The average power of the m th element in $\mathbf{d}_n$ can be also found from (7) as

$$\sigma_x^2 \sum_{l=0}^{M-1} \left|C[lN+n]H[lN+n]u_{l,m}\right|^2 + \sigma_V^2 \sum_{l=0}^{M-1} \left|C[lN+n]u_{l,m}\right|^2$$
$$= q_1[n,m]. \quad (9)$$

Therefore, the output SNR after equalization can be expressed as

$$\gamma[n,m] = \frac{q_0[n,m]}{q_1[n,m] - q_0[n,m]}. \quad (10)$$

According to the MMSE criterion, $\mathbf{C}_n$ should be designed so that

$$\begin{aligned} &E\{(\mathbf{d}_n - \mathbf{x}_n)'(\mathbf{d}_n - \mathbf{x}_n)\} \\ &= E\{(\mathbf{U}\mathbf{d}_n - \mathbf{U}\mathbf{x}_n)'(\mathbf{U}\mathbf{d}_n - \mathbf{U}\mathbf{x}_n)\} \\ &= E\{(\mathbf{C}_n\mathbf{R}_n - \mathbf{U}\mathbf{x}_n)'(\mathbf{C}_n\mathbf{R}_n - \mathbf{U}\mathbf{x}_n)\} \end{aligned} \quad (11)$$

is minimized. Using the orthogonality principle, we have

$$E\{(\mathbf{C}_n\mathbf{R}_n - \mathbf{U}\mathbf{x}_n)\mathbf{R}_n'\} = \mathbf{0} \quad (12)$$

and consequently,

$$\begin{aligned} \mathbf{C}_n &= E\{\mathbf{U}\mathbf{x}_n\mathbf{R}_n'\}(E\{\mathbf{R}_n\mathbf{R}_n'\})^{-1} \\ &= \mathbf{U}E\{\mathbf{x}_n\mathbf{x}_n'\}\mathbf{U}'\mathbf{H}_n'(\mathbf{H}_n\mathbf{U}E\{\mathbf{x}_n\mathbf{x}_n'\}\mathbf{U}'\mathbf{H}_n' + E\{\mathbf{V}_n\mathbf{V}_n'\})^{-1} \\ &= \mathbf{H}_n'\left(\mathbf{H}_n\mathbf{H}_n' + \frac{1}{\gamma_{in}}\mathbf{I}\right)^{-1} \end{aligned} \quad (13)$$

where $\gamma_{in} = \frac{\sigma_x^2}{\sigma_V^2}$ is the input SNR before equalization.

From (13), the diagonal element is found to be

$$C[lN+n] = \frac{H^*[lN+n]}{\left|H[lN+n]\right|^2 + \frac{1}{\gamma_{in}}}. \quad (14)$$

Substituting (14) into (8) and (9) and using (10), the output SNR after MMSE equalization is finally expressed as

$$\gamma[n,m] = \frac{\sum_{l=0}^{M-1} \frac{\left|H[lN+n]u_{l,m}\right|^2}{\left|H[lN+n]\right|^2 + \frac{1}{\gamma_{in}}}}{1 - \sum_{l=0}^{M-1} \frac{\left|H[lN+n]u_{l,m}\right|^2}{\left|H[lN+n]\right|^2 + \frac{1}{\gamma_{in}}}}. \quad (15)$$

If we only consider a class of unitary matrices satisfying $\left|u_{l,m}\right|^2 = \frac{1}{M}$, such as those adapted from Fourier transform matrix , Hadamard matrix and rotated Hadamard matrix [19], (15) can be simplified as

$$\gamma[n] = \frac{\frac{1}{M}\sum_{l=0}^{M-1} \frac{\left|H[lN+n]\right|^2}{\left|H[lN+n]\right|^2 + \frac{1}{\gamma_{in}}}}{1 - \frac{1}{M}\sum_{l=0}^{M-1} \frac{\left|H[lN+n]\right|^2}{\left|H[lN+n]\right|^2 + \frac{1}{\gamma_{in}}}} = \frac{1}{\frac{1}{M}\sum_{l=0}^{M-1} \frac{1}{\left|H[lN+n]\right|^2\gamma_{in} + 1}} - 1 \quad (16)$$

We see that the output SNR is determined by the channel frequency response $H[k]$, or equivalently, the channel impulse response $h[i]$. Assuming QPSK modulation for data symbols and making a Gaussian distribution approximation for ISI, the bit error probability of the equalizer for a realization of the channel impulse response can be evaluated as $\frac{1}{N}\sum_{n=0}^{N-1} Q\left(\sqrt{\gamma[n]}\right)$, where the Q-function is defined as $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{t^2}{2}} dt$. Also, assuming that the channel impulse response has L independent paths, each of which is modelled as an independent complex Gaussian process, the average BER for such frequency-selective fading channel can be evaluated as

$$P_e(M,L) = E_h\left\{\frac{1}{N}\sum_{n=0}^{N-1} Q\left(\sqrt{\gamma[n]}\right)\right\} \tag{17}$$

where $E_h\{\cdot\}$ denotes the ensemble averaging over all possible $h[i]$. We see that (17) is a function of the data group size M and the multipath length L.

IV. BER LOWER BOUNDS AND APPLICATION

To show the relationship between the system performance and the group size as well as the relationship between the system performance and the channel diversity order, let's work out two sets of BER lower bounds using the MMSE equalization. The first set represents the best possible performance for a given block size M. We assume that the channel provides a full multipath diversity, i.e., $L >> M$, so that $H[lN+n]$ at different l s become independent complex Gaussian variables with unit variance and are alternatively denoted as α_l for convenience. Then, the average BER can be alternatively evaluated as

$$P_e(M,\infty) = E_\alpha\left\{Q\left(\sqrt{\frac{1}{\frac{1}{M}\sum_{l=0}^{M-1}\frac{1}{\gamma_{in}|\alpha_l|^2+1}}-1}\right)\right\} \tag{18}$$

where $E_\alpha\{\cdot\}$ denotes the ensemble average over α_0, α_1,..., α_{M-1}. In (18), we can also express γ_{in} as $2\frac{E_b}{N_0}$ for QPSK, where E_b is the signal energy per bit and N_0 is the noise power spectral density.

Fig. 2 shows the lower bounds of the MMSE equalization performance under different block sizes M =1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, and ∞ by the Monte Carlo method, i.e., instead of evaluating (18) using the joint probability density function (pdf) of α_0, α_1,..., α_{M-1}, we generate sufficient realizations of these independent complex Gaussian variables, evaluate the BER for each realization, and then take an average. We see that the performance is improved as M increases. When $M \to \infty$, since $|\alpha_l|^2$ is chi-square-distributed with two degrees of freedom and pdf $e^{-\rho}$, a closed-form lower bound expression can be found as

$$P_e(\infty,\infty) = Q\left(\sqrt{\frac{1}{\int_0^\infty \frac{e^{-\rho}}{\gamma_{in}\rho+1}d\rho}-1}\right) \tag{19}$$

which is the best performance the MMSE equalization could offer.

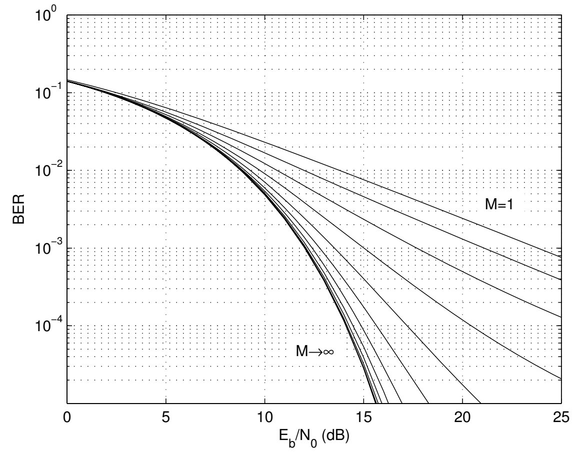

Fig. 2. Lower bounds of MMSE equalization performance under different data group sizes. The curves from right to left correspond to M=1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, and ∞ respectively.

The second set of lower bounds indicates the best performance for a given number of channel multipath L (referred to as multipath diversity order) with sufficiently large data block size M. Let $M \to \infty$, the average BER can be evaluated as

$$P_e(\infty,L) = E_h\left\{Q\left(\sqrt{\frac{1}{\frac{1}{2\pi}\int_0^{2\pi}\frac{1}{\gamma_{in}\left|H\left(e^{j\omega}\right)\right|^2+1}d\omega}-1}\right)\right\} \tag{20}$$

where $H\left(e^{j\omega}\right)$ is the Fourier transform of $h[i]$. Fig. 3 shows this set of lower bounds for L=1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, and ∞ by the Monte Carlo method, i.e., we generate sufficient realizations of the channel $h[i]$, evaluate BER for each realization, and then take an average. We see that as the channel diversity order increases the performance is also improved. When $L \to \infty$, the average BER approaches the same best performance expressed by the closed-form equation (19).

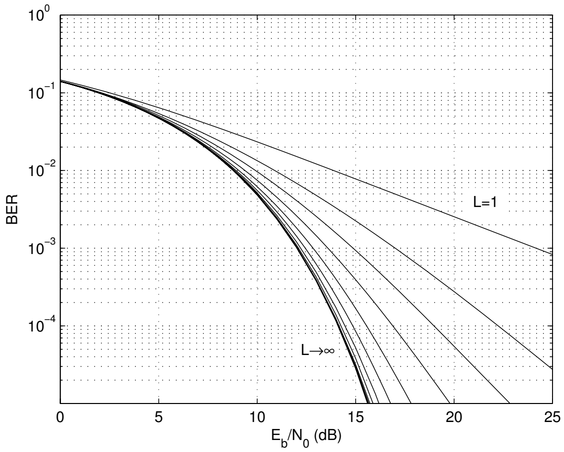

Fig. 3. Lower bounds of MMSE equalization performance under different multipath diversity orders. The curves from right to left correspond to L=1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, and ∞ respectively.

Comparing Fig. 2 with Fig. 3, we notice that $P_e(1,\infty) = P_e(\infty,1)$, i.e., when there is no preccoding (M=1), the best performance a conventional OFDM system can achieve is the same as the performance with diversity order one (L=1). This is consistent with what we have known about the conventional OFDM. When $1 < M = L < \infty$, we have $P_e(M,\infty) > P_e(\infty, L)$. This means that for a given data group size M the precoded OFDM system can not achieve the performance which the system could potentially offer with diversity order $L = M$. However, as M becomes larger, the performance gradually approaches the best performance that the system provides for a given diversity order.

To confirm the above evaluated BER lower bounds and also demonstrate their usefulness, the performance for a precoded OFDM system with MN=128 is tested by numerical simulation using a precoding matrix adapted from Fourier transform matrix under channel multipath diversity order L=32. Fig. 4 shows the performance using MMSE equalization for different group sizes M=1, 2, 4, 8, 16, 32, 64, and 128 (consequently N=128, 64, 32, 16, 8, 4, 2, and 1 respectively). We see that when $M \le 32$ (i.e., the data group size is less than or equal to the multipath diversity order), the performance agrees with the lower bound $P_e(M,\infty)$ for the given M. When choosing the maximum data group size M=128 (i.e., N=1), the performance approaches the lower bound $P_e(\infty, L)$ for L=32. Also note that when M=64 (i.e., N=2), the performance is already very close to this lower bound.

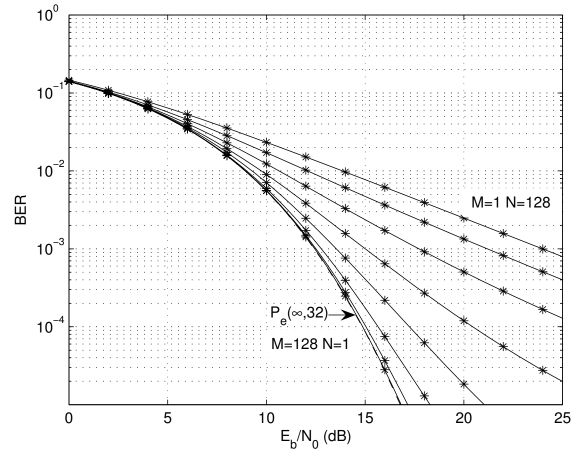

Fig. 4. Performance of MMSE equalization for precoded OFDM system with MN=128 under channel diversity order L=32 and different group sizes M=1, 2, 4, 8, 16, 32, 64, and 128 (consequently N=128, 64, 32, 16, 8, 4, 2, and 1 respectively). Dashed line is the lower bound for L=32.

The above example shows that we can use the evaluated lower bounds to determine the data group size for a proper precoded OFDM system design. First, from $P_e(\infty, L)$ we can predict the best performance the system could potentially offer once the multipath diversity order is given. Then, from $P_e(M,\infty)$ we can compare system performance under different precoding sizes and decide a suitable M subject to some complexity constraint. We can also estimate the performance degradation for a chosen M. Applying the above guidelines to a practical system, the multiband (MB) OFDM for ultra-wideband applications [4], it is of interest to reveal that the system design seems inappropriate regarding the dual-carrier modulation (DCM) for data rate over 320 Mbps. The MB-OFDM system uses 128 subcarriers with a ZP of length 32 which corresponds to a potential multipath diversity of order 32. Since the DCM is equivalent to a precoding with only group size two, it is easily seen that the multipath diversity potential is not fully exploited.

V. CONCLUSIONS

We have shown that the performance of the precoded OFDM systems using MMSE equalization is determined by the precoding data group size and the diversity order that the multipath channel can provide. Two sets of BER lower bounds are evaluated by the Monte Carlo method assuming full diversity order and sufficiently large group size respectively. A closed-form lower bound expression is also derived to define the theoretical performance limit for the MMSE equalization. These lower bounds can serve as the guidelines for precoded OFDM system design.

References

[1] S. B. Weinstein and P. M. Ebert, "Data transmission by frequency-division multiplexing using the discrete Fourier transform," *IEEE Transactions on Communication Technology*, COM-19, October 1971, pp. 628–634.

[2] J. A. C. Bingham, "Multicarrier modulation for data transmission: An idea whose time has come," *IEEE Communications Magazine*, Vol. 28, May 1990, pp. 4-14.

[3] IEEE Standard 802.11g/D1.0, "Wireless LAN medium access control (MAC) and physical layer (PHY) specifications: further higher-speed physical layer extension in the 2.4 GHz band," November 2001.

[4] WiMedia Alliance, "MultiBand OFDM physical layer specification," Release 1.1, July 2005.

[5] IEEE 802.16/D5, "Draft IEEE standard for local and metropolitan area networks – Part 16: Air interface for fixed broadband wireless access systems," May 2004.

[6] U. Reimers, "Digital video broadcasting," *IEEE Communications Magazine*, Vol. 36, No. 6, June 1998, pp. 104-110.

[7] 3GPP TR25.814/V7.0.0, "3rd generation partnership project technical specification group radio access network physical layer aspect for evolved universal terrestrial radio access (UTRA) (Release 7)", June 2006.

[8] A. E. Jones, T. A. Wilkinson, and S. K. Barton, "Block coding scheme for reduction of peak-to-mean envelope power ratio of multicarrier transmission schemes," *Electronics Letters*, Vol. 30, No. 25, December 1994, pp. 2098–2099.

[9] P. Van Eetvelt, G. Wade, and M. Tomlinson, "Peak-to-average power reduction for OFDM schemes by selective scrambling," *Electronics Letters*, Vol. 32, October 1996, pp. 1963–1964.

[10] K. G. Paterson, "Generalized Reed–Muller codes and power control in OFDM modulation," *IEEE Transactions on Information Theory*, Vol. 46, January 2000, pp. 104–120.

[11] L. J. Cimini, Jr. and N. R. Sollenberger, "Peak-to-average power ratio reduction of an OFDM signal using partial transmit sequences," *IEEE Communications Letters*, Vol. 4, March 2000, pp. 86–88.

[12] T. Pollet, M. Ven Bladel, and M. Moeneclaey, "BER sensitivity of OFDM systems to carrier frequency offset and Wiener phase noise," *IEEE Transaction on Communications*, Vol. 43, February/March/April 1995, pp 191-193.

[13] M. Speth, S. Fechtel, G. Fock, and H. Meyr, "Optimal receiver design for wireless broad-band systems using OFDM, Part I," *IEEE Transactions on Communications*, Vol. 47, No. 11, November 1999, pp. 1668-1677.

[14] C. Berrou, A. Glavieux, and P. Thitimajshima, "Near Shannon limit error-correcting coding and decoding: Turbo-codes," in *Proceedings of IEEE International Conference on Communications*, Geneva, Switzerland, May 1993, Vol. 2, pp. 1064-1070.

[15] R. G. Gallager, "Low-density parity-check codes," *IEEE Transactions on Information Theory*, Vol. 8, No. 1, January 1962, pp. 21-28.

[16] L. J. Cimini, Jr., "Analysis and simulation of a digital mobile channel using orthogonal frequency-division multiplexing," *IEEE Transactions on Communications*, Vol. 33, July 1985, pp. 665–675.

[17] Z. Wang and G. B. Giannakis, "Linearly precoded or coded OFDM against wireless channel fades?," in *Proceedings of Signal Processing Advances in Wireless Communications Workshop*, Taoyuan, Taiwan, March 20–23, 2001, pp. 267–270.

[18] A. Bury, J. Engle, and J. Linder, "Diversity comparison of spreading transforms for multicarrier spread spectrum transmission," *IEEE Transactions on Communications*, Vol. 51, No. 5, May 2003, pp.774-781.

[19] M. L. McCloud, "Optimal binary spreading for block OFDM on multipath fading channels," in *Proceedings of IEEE Wireless Communications and Networking Conference*, Atlanta, GA, March 2004, pp.965-970.

[20] Z. Liu, Y. Xin, and G. B. Giannakis, "Linear constellation precoding for OFDM with maximum multipath diversity and coding gains," *IEEE Transactions on Communications*, Vol. 51, No. 3, March 2003, pp. 416-427.

[21] M. L. McCloud, "Analysis and design of short block OFDM spreading matrices for use on multipath fading channels," *IEEE Transactions on Communications*, Vol. 53, No. 4, April 2005, pp. 656-665.

[22] K. I. Ahmed, C. Tepedelenlioglu, and A. Spanias, "Performance of precoded OFDM with channel estimation error," *IEEE Transactions on Signal Processing*, Vol. 54, No. 3, March 2006, pp. 1165-1171.